Dynamical chiral symmetry breaking, asymptotic freedom and absence and presence of fermion pole(s)

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Dynamical mass generation is studied in the framework of Schwinger-Dyson equations. For this purpose the fermion propagator is studied for massless and massive vector theory and it is found that when the asymptotic freedom is incorporated into the Fukuda-Kugo equation the modeled quark propagator has at least one real pole in the timelike region. General property of the solutions for quark propagator in the Minkowski space is further discussed.

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I. INTRODUCTION

Theoretical analysis of the chiral symmetry breaking in QCD and related impact to the low energy processes remains a long-standing and still not completed challenge. While the discovery of asymptotic freedom [1] has opened the save gate for perturbation theory use for high energy processes studies, the consistent description of strong interaction in the infrared region remains so far less explored subject. The lattice calculation represents the most common nonperturbative Euclidean space technique available for this purpose. Using this, then the timelike part of the Minkowski space is the dark side of the moon for the Earth lattice community. However, although not so much explored in practise, a natural nonperturbative framework for the study of infrared properties of QCD Green's functions in the whole Minkowski space is the formalism of Schwinger-Dyson equations (SDEs).

The complex of SDEs is an infinite tower of coupled integral equations which must be truncated to be tractable in practice. Some studies have concerned on the solution of pure gluodynamics [2, 3] others included the SDE for quark propagator [4] thus including also the fermion loops into the gluon vacuum self-energy. In recent, there is a notable agreement between the recent Euclidean space solutions of SDEs and the lattice results. For a review of recent progress see [4]. The formalism of SDEs provides continuous connection between the physics of large and small lengths. Although at the first look the ultraviolet and infrared physics can seem to be largely unrelated, the meaning of asymptotic freedom (high momentum scale phenomena) for dynamical chiral symmetry breaking (supposedly infrared phenomena) has been recognized. In the pioneer study [5] the asymptotic freedom has been implemented by the running coupling which behaves like inverse of power of momenta in the ultraviolet. Many realistic studies considered lately has included more correct log behaviour of the running coupling in various approximation of quark gap equation. The asymptotic freedom and its associated regulating character of the ultraviolet modes is the necessary condition which make chiral symmetry breaking physically meaningful in QCD, where it is visualized by the actual regularization scheme independent calculations [4] (SDEs), [6] (functional renormalization group) and similar strong coupling field theories (see [7] for Technicolor studies).

In fact, hitherto almost the all nonperturbative results have been achieved in the Euclidean space formalism ,i.e. the Wick rotation switches formally to the imaginary time formalism, or the Euclidean space formalism is used as a starting point (even when the Wick rotation is prohibited, for this point see the discussion in [9]). The primary objective of this paper is to find a solutions for the quark propagator functions in the whole Minkowski space. To this point we should mention a certain progress in Minkowski solution of dynamical chiral symmetry breaking by using a dispersion relation technique [10, 11]. Apparently these methods provide a reliable solutions in the vicinity of the critical coupling, its adequacy for description of real QCD has been questioned only very recently. This is just a huge increment of dynamical light quarks mass in the infrared which form one side gives rise the assumed value constituent quark masses which exceed the scale $\Lambda_{QCD} \simeq 200MeV$ but from the other side makes the use of dispersion technique more complicated and less trustworthy when pronounced in practise. In this paper we avoid possible weaknesses accompanying the analytical constraint of dispersion/spectral relations and extend the technique of Fukuda-Kugo solution [12] of ladder fermion SDE to the case when the asymptotic freedom is consistently incorporated. For this purpose we exploit the property of the analytic running coupling [13]. Thus, although we do not assume the standard analyticity for quark propagator, we do assume this for the gluon propagator, or more precisely pronounced, for the associate running coupling . Although such analyticized running coupling in used is not a direct observable and its relation with QCD effective charges (observables in the sense of Grungerg [14]) is not rigorously established, it appears to be very useful tool for our Minkowski space study. The dynamical chiral symmetry breaking with analytic invariant QCD charge has been already studied in the Euclidean space by Papavasilliou and Nesterenko in [15]. For a recent review of running coupling(s) in QCD see [16].

The layout of the paper is as follows. Section II is devoted to the overview of Fukuda-Kugo (timelike) ladder fermion SDE with hard cutoff regularization. The results with massless and massive gauge boson propagators inside the loop of SDE are obtained by the direct solution of the original integral equation. The absence of fermion poles has been actually confirmed in the region predicted by the authors of [12], however, in the light of our further discussion we differ in the interpretation. In Section III we discus the analytic invariant running coupling constant and write down the quark SDE in generalized ladder SDE. Since the form of analytical running coupling allows us to perform the angular integration analytically, we consequently make continuation to the timelike axis of square of four-momenta. The numerical solution is provided in Section IV in the approximation wherein the weight function of analytical charge is replaced by the delta function. Contrary to the naively regularized ladder QED, the real fermion pole has been always identified. It includes very strong coupling and vanishing boson mass cases as well as. In Section V we discuss the possible form of singularities in the context of Lorentz invariance constraint. In Conclusion (Section VI) the basic results are summarized and further directions of research within this approach are outlined.

In this paper we will use the following conventions: the positive variables x, y will represent the square of momenta in the whole Minkowski space, i.e. for instance $x = p^2$ for timelike momenta when $p^2 > 0$ while $x = -p^2$ for p^2 in spacelike region. Note, our metric is $g_{\mu\nu} = diag(1, -1, -1, -1)$. For purpose of clarity we label the mass function B as B_s in the spacelike region of fourmomenta and as B_t when evaluated for timelike fourmomentum (i.e. $B(p^2) = B_s(x)\theta(-p^2) + B_t(x)\theta(p^2)$).

II. MASKAWA-NAKAJIMA/FUKUDA-KUGO EQUATION

In this Section we discuss a very simple approximation of fermion SDE: the Maskawa-Nakajima/Fukuda-Kugo equation. It represents ladder approximation of fermion SDE in some gauge theory with spin one gauge boson and it does not incorporate the running of the coupling since the boson selfenergy and corrections to the vertex are simply omitted. Furthermore, and in fact due to this omission, the momentum integration in gap equation has been regulated by an upper boundary cutoff.

In parity conserving theory the fermion propagator S can be characterized by two independent scalar function A, B such that $S(p) = [A \not{p} - B]^{-1}$ (bare fermion propagator is $S_0 = [\not{p} - m_0]^{-1}$). In the ladder approximation, the equation for inverse of S can be written

$$\not{p}A(p^2) - B(p^2) = \not{p} - m_0 - ig^2 \int \frac{d^4q}{(2\pi)^4} \gamma_\alpha G^{\alpha\beta}(p-q) S(q) \gamma_\beta \tag{2.1}$$

where m_0 is a bare mass and $G^{\alpha\beta}$ is boson propagator (for detailed derivation of set of SDE's see for instance [9] or [17]). The equation Eq. (2.1) and the classification of the solution has been discussed in [18] especially in Landau-like gauge for which the massive propagator reads

$$G^{\alpha\beta}(q) = \frac{-g^{\alpha\beta} + \frac{q^\alpha q^\beta}{q^2}}{q^2 - \mu^2 + i\varepsilon} \, . \tag{2.2}$$

Few years later, Fukuda and Kugo have found the solution for the timelike momenta for $\mu = 0$. They observe no real pole for resulting fermion propagator. Hence the absence of free propagating mode has been interpreted as a sign for confinement at that time. First, disregarding the known deficiency of integral cutoff regularization scheme, we overview the method of solution and resolve (2.1) also for nonzero μ. Up to our knowledge such solution has never been explicitly published in the literature. We leave the question of the reliability of 'would be' confining solution into the discussing section IV..

In the next we will follow the paper [12], using $A = 1$ approximation and making Wick rotation and angular integration (see Appendix A) we get

$$\begin{aligned} B_s(x) &= m_0 + \frac{3C}{4} \int_0^{\Lambda^2} dy \, \frac{B_s(y)}{y + B_s^2(y)} K(x, y, \mu^2) \\ K(x, y, \mu^2) &= \frac{2y}{x + y + \mu^2 + \sqrt{(x + y + \mu^2)^2 - 4xy}} \end{aligned} \tag{2.3}$$

where Λ is the cutoff and where $C = 3g^2 C_2(R)/4\pi^2$, $C_2(R)$ denotes the Casimir invariant of the quark representation ($C_2(R) = 4/3$ for QCD). The equation (2.3) can be quite easily solved by the method of iteration avoiding thus inconvenient conversion to the nonlinear differential equation. When $m_0 = 0$ but $B \neq 0$ we talk about dynamical chiral symmetry breaking.

Having kept the solution $B_s(y)$ for spacelike y we can define the 'synthetic fermion mass' $\hat{B}(x)$ at timelike axis of fourmomenta such that $\hat{B}(x) = B_s(-x)$ is a solution of integral (2.3) for timelike x:

$$\hat{B}(x) = m_0 + \frac{3C}{4}\int_0^{\Lambda^2} dy\, \frac{B_s(y)}{y + B_s^2(y)} K(-x,y,\mu)$$
$$K(-x,y,\mu) = \frac{2y}{-x + y + \mu^2 + \sqrt{-x+y+\mu^2)^2 + 4xy}}\,. \tag{2.4}$$

which represents correct continuation until the first singularity is crossed on the timelike axis of p^2. The continuation on the usual physical cut of timelike momenta $x > (m+\mu)^2$ reads

$$B_t(x) = m_0 + \hat{B}(x) - \frac{3C}{4}\int_0^{(\sqrt{x}-\mu)^2} dy\, \frac{B_t(y)}{y - B_t^2(y) + i\varepsilon} X(x;y,\mu^2) \tag{2.5}$$

where $X(x;y,\mu^2)$ is the discontinuity of $K(-x,y,\mu^2)$ over the physical cut:

$$X(x;y,\mu^2) = \frac{\sqrt{(-x-y+\mu^2)^2 - 4xy}}{x}\,, \tag{2.6}$$

and note the presence of Feynman infinitesimal $i\varepsilon$, which has been omitted in the original paper [12].

To see this continuation was correct let us assume the smoothness of B at the vicinity of the pole $y = m^2$, i.e. $B_t^2(m^2) = m^2$. Then one can use the functional relation

$$\frac{1}{\mathcal{O} \pm i\epsilon} = P.\frac{1}{\mathcal{O}} \mp i\pi\delta(\mathcal{O}) \tag{2.7}$$

and obtain the dominant contribution to the absorptive part of mass function B

$$\mathbf{Im}B(x) = \frac{3C}{4} m\pi X(x;m^2,\mu^2)\Theta(x - (\mu+m)^2)\,, \tag{2.8}$$

where Θ stands for the standard Heaviside step function. Clearly, (2.8) represents one loop perturbation theory result when C is a small parameter, $3C/4 << 1$.

In general one have to solve the Eq. (2.5). Assuming smoothness of the function B the equation (2.5) can be rewritten as follows:

$$B_t(x) \;=\; m_0 + \hat{B}(x) - i\frac{3C}{4}\pi\sum_j \frac{m_j}{|1 - 2B_tB_t'|_j} X(x;m_j^2,\mu^2)\Theta(x - (\mu+m_j)^2) - I(x) \tag{2.9}$$

$$I(x) \;=\; \frac{3C}{4}P.\int_0^{(\sqrt{x}-\mu)^2} dy\, \frac{B_t(y)}{y - B_t^2(y)} X(x;y,\mu^2)\Theta(x-\mu^2) \tag{2.10}$$

where j runs over the roots of Eq. $y - B_t^2(y) = 0$ and where we have used the following abbreviation:

$$|1 - 2B_tB_t'|_j = \left|1 - 2B_t(y)\frac{dB_t(y)}{dy}\right|_{y_j=m_j^2} = \left|1 - \frac{dB_t(y)}{d(y^{1/2})}\right|_{y_j=m_j^2}\,. \tag{2.11}$$

The Eq. (2.9) represents integral inhomogeneous equations (even for zero m_0) with the one singular kernel in the integrand of I. It can be solved by a standard numerical method and we provide some details in the Appendix B. The dynamical chiral symmetry breaking is of great interest for us and we will concern on these solutions in this paper. Inclusion of small explicit chiral breaking term is straightforward and we leave this case aside of our interest.

In order to scale the dimensionfull quantities we take $\mu = 1$ in arbitrary units. Further, the numerical hard cutoff is adjusted to be $\Lambda = e^5\mu \simeq 148\mu$. Its value constraints the meaningful region of the square of fourmomenta as $p^2 \in (-\Lambda^2, \Lambda^2)$. The numerical solutions for various coupling constant are presented in Fig. 1.

As expected already from the discussion in [12] there are several phases of the theory. They have been confirmed and actually found also here. They are as follows:

I. Chiral symmetric phase- for the coupling bellow the critical value α_c, there is only trivial solution $B = 0$.

II. Chiral symmetry breaking phase, where the propagator has a real pole. In this case we get also nontrivial solution for B. For our parameters μ, Λ this phase is characterized by the coupling strength $\alpha \in (\alpha_c, 1.4)$ and the pole

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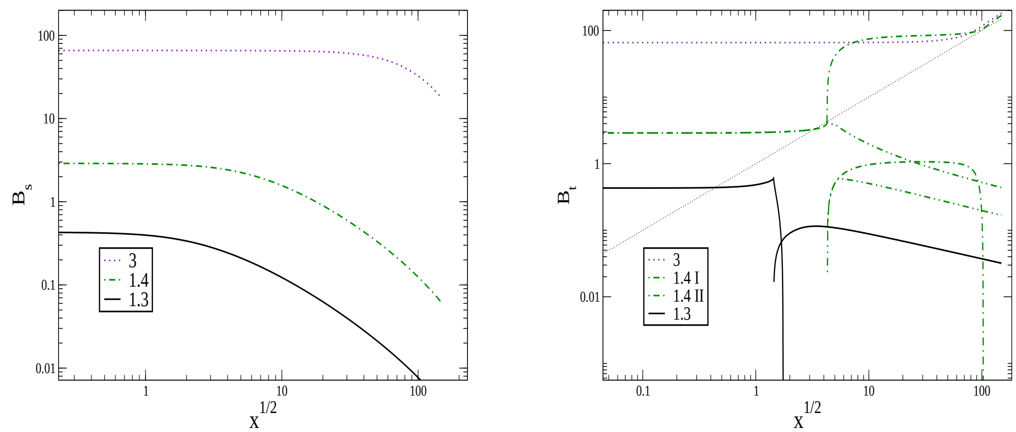

FIG. 1: Dynamical mass function for spacelike (left) and timelike (right) region of fourmomenta for massive boson case and hard UV cutoff. The solutions are labeled by $\alpha = g^2/4\pi$. On the right panel the lines representing the imaginary part of B becomes nonzero above the threshold. Notice, there are two solutions for the coupling $\alpha = 1.4$. The pole mass are identified by the crossing of thin dotted line (the function $\sqrt(p^2)$).

mass m is at most of order μ. In fact, we can observe and have found two solution for $\alpha = 1.4$. Although numerically they both cut the axis $\sqrt{p^2}$, the one of them only sweep the region under the diagonal and their timelike high energy asymptotic crucially differ. Of course, the question of uniqueness of such continuation naturally arises.

III. Chiral symmetry breaking phase above $\alpha = 1.4$, where the mass ratio $m/\mu \simeq 4.2$ (and $B(0) \simeq 3\mu$) is achieved, the pole and associate analytical cut vanish and the mass function blows to 'infinity' like $\sqrt(p^2)$ for large p^2. Note, no sign of such transition is observed in the spacelike regime. Clearly the spacelike solution does not reflect dramatic changes that happen in timelike part of the Minkowski space. The solutions in the regime III. have been originally called 'confining' because of the absence of real pole and associate particle production threshold. In fact we have to be more careful with the interpretation, particularly because the Wick rotation is not allowed, the rotation of the contour cross the singularity at timelike momentum infinity. Clearly, in this case the hard cutoff regularization of the momentum integral does not commute with the Wick rotation and there is neither confidence that we obtain a true solution in the whole Minkowski space. This is because we are dealing with non-asymptotic free theory where the result are not independent on the UV integral cutoff. We argue here, the "would be" confining solution would be meaningful only when the theory is defined in the Euclidean space from the beginning. Clearly, this is not the case of ladder QED and the observed nontrivial solution can be an artifact of given scheme. Although we suppose that these things are rather well known, we discussed them explicitly here for the purpose of clarity.

A. Massless vector boson

The solution for massless vector boson has been firstly obtain in [12]. For this purpose the authors of the paper [12] transform the integral equation into the differential one which has been solved numerically. Here we confirm their solution by direct solution of the original integral equation. Taking a limit $\mu \to 0$ in Eq's (2.3) and (2.9) is straightforward. The SDE for selfenergy in the massless case ($m_0 = \mu = 0$) then reads

$$B_s(x) = \int_0^x dy \left(\frac{y}{x} - 1\right) \frac{B_s(y)}{y + B_s^2(y)} + \frac{< \psi\bar{\psi} >_\Lambda}{N_c} \tag{2.12}$$

$$B_t(x) = \frac{< \psi\bar{\psi} >_\Lambda}{N_c} - \frac{3C}{4} \int_0^x dy \left(1 - \frac{y}{x}\right) \frac{B_t(y)}{y - B_t^2(y)} \tag{2.13}$$

where N_c is the number of colors and where we have already omitted intensifimal imaginary prefactor $i\varepsilon$ since as it is known that the equation $y - B_t^2(y) = 0$ has no real roots in hard cutoff theory. The inhomogeneous term in (2.13)

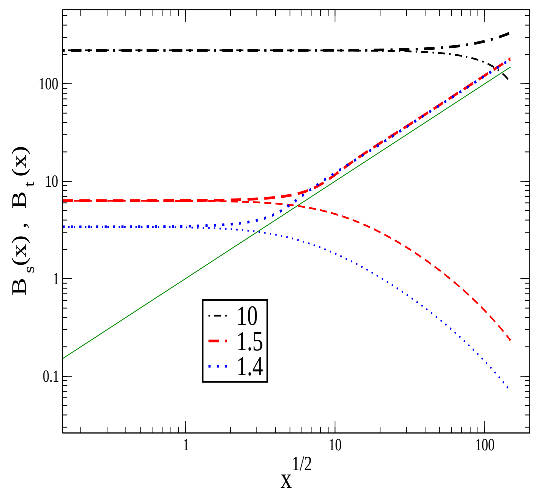

FIG. 2: Fermion dynamical mass function in ladder approximation with hard cutoff $\Lambda^2 = e^{10}$ (in arbitrary units) for the three value of the coupling constant $\alpha = 1.4, 1.5, 10$. The thin dotted line represents $\sqrt{p^2}$, the solution for timelike momenta are labeled by thick 'upper' lines, the spacelike solutions are added for the comparison (decreasing lines of the same type belongs to the same coupling, clearly B_s and B_t are identical for $x = p^2 = 0$)

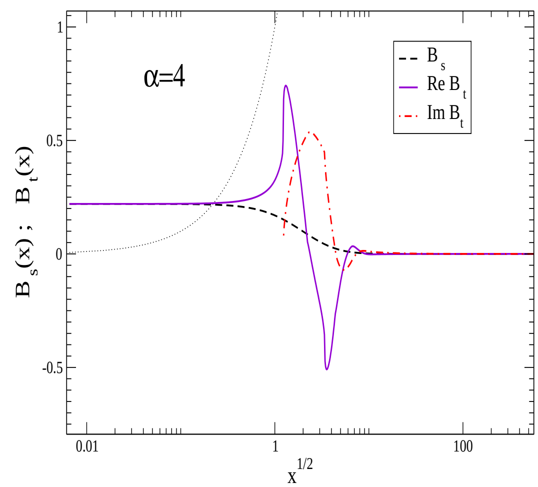

FIG. 3: Dynamical quark mass for $\alpha^* = 4$ as described in the text.

is simple constant-the usual fermion condensate:

$$\hat{B} = \frac{< \psi\bar{\psi} >_\Lambda}{N_c} = \frac{3C}{4}\int_0^{\Lambda^2} \frac{B_s(y)}{y + B_s^2(y)} \tag{2.14}$$

The numerical results are shown in Fig. 2 for three distinct values of the coupling constant. Similarly to previously discussed region III of massive boson case the results could be interpreted with a great care. Again the Wick rotation invalidates because of naive regularization scheme and the resulting timelike high momentum behaviour can be an artifact of inappropriate calculation scheme.

III. QUARK MASS GENERATION WITH ANALYTICIZED RUNNING COUPLING APPROXIMATION

The construction of quark gap equation within the analytical QCD running coupling has been already studied in [15] where interested reader can find the motivations and some further details. For a compact explanation we mention basic points also here.

The strategy of the usage of analyticized QCD running coupling relies on the following prescription of the SDE kernel:

$$g^2 G^{\mu\nu}(k)\Gamma_\nu(q,p) \rightarrow \left[-g^{\mu\nu} + \frac{k^\mu k^\nu}{k^2}\right] \frac{4\pi\alpha(k^2,\Lambda)}{k^2-\mu^2+i\varepsilon}\gamma_\nu \tag{3.1}$$

where, inspired by the effective running charge, the analytical running coupling is written via dispersion relation as

$$\alpha(q^2,\Lambda) = \int_0^\infty d\lambda \, \frac{\rho_g(\lambda,\Lambda)}{q^2-\lambda+i\varepsilon} \tag{3.2}$$

and where we have also introduced effective gluon mass μ. Recall, the mass parameter μ has nothing to do with the observable mass of free particle since gluon should be confined. Nevertheless we keep this parameter nonzero to model possible dynamical gluon mass generation [3, 19]. For the phenomenological impact of massive gluons see [20]. Note, contrary to the usual definition of effective QCD charges [14, 21], the analytical running coupling here is not a direct observable and should be understood as so. The spectral function ρ_g is a regular function and ensure the regularity of the running coupling in the whole complex plane apart the real timelike positive semi-axis. In the one loop approximation it can be taken as following

$$\rho_g(\lambda,\Lambda)=\frac{4\pi/\beta}{\pi^2-\ln\left(\lambda^2/\Lambda^2\right)}+... \tag{3.3}$$

where $\Lambda=\Lambda_{QCD}\simeq 200MeV$ for two quark flavor and the dots represent the ambiguity stemming from the missing knowledge of infrared nonperturbative contributions in 'low λ' modes. Here we only assume that such contribution does not affect $\log q^2$ behaviour of α at UV region, i.e. for $q^2>>\Lambda^2$. Recall here also that in the approximation (3.1) we have ignored all tensorial structure of the vertex Γ but not γ_μ. The neglected pieces of longitudinal part and the transverse part of the full vertex Γ can be naturally included by consideration of two skeleton loop contribution in Σ.

In addition we assume no singularities of the quark propagator in the first and the third quadrant of complex p^2 plane. Then we see that the kernel of the quark SDE allows to perform standard Wick rotation without cutting any singularities. The are two meaningful advantages of presented method: one is that we are able to avoid angle approximation usually performed in the Euclidean space, the second is that the resulting Euclidean solution can be quite easily continued on the timelike p^2 axis exactly as in the case of Fukuda-Kugo equation. However here, the log behaviour of the running coupling sufficiently regularize the kernel of SDE . Contrary to the cases studied in the previous Section, the quark gap equation with zero bare mass does not require any additional regularization.

In what follows we are going to write down gap equation in the whole Minkowski space. Further, we simplify the system of equation by making $A=1$ approximation, after the Wick rotation one can arrive to the following quark SDE:

$$B_s(x)=m_0+16\int\frac{d^4q}{\pi^4}\frac{\pi\alpha(z)}{z+\mu^2}\frac{B_s(y)}{y+B_s^2(y)} \tag{3.4}$$

where $x=-p^2, y=-q^2, z=-(p-q)^2$. Taking the integral representation (3.2) and using a simple algebra we get

$$B_s(x)=m_0+\int_0^\infty d\lambda\int\frac{d^4q}{\pi^4}\frac{16\pi\rho(\lambda)}{\lambda-\mu^2}\left[\frac{1}{z+\mu^2}-\frac{1}{z+\lambda}\right]\frac{B_s(y)}{y+B_s^2(y)}\,. \tag{3.5}$$

Performing the angular integration the resulting equation for B can be cast into the form:

$$\begin{aligned}
B_s(x)&=m_0+\frac{1}{\pi}\int_0^\infty dy\,\frac{B_s(y)}{y+B_s^2(y)}K(x,y)\\
K(x,y)&=\int_0^\infty d\lambda\frac{\rho(\lambda)}{\lambda-\mu^2}\left[K(x,y,\mu^2)-K(x,y,\lambda)\right]\\
&=\int_0^\infty d\lambda\frac{\rho(\lambda)}{2x(\lambda-\mu^2)}\{\mu^2-\lambda^2-\sqrt{(x+y+\mu^2)^2-4xy}+\sqrt{(x+y+\lambda^2)^2-4xy}\}
\end{aligned} \tag{3.6}$$

In order to perform the continuation to the timelike axis we proceed similar steps as in the case of Fukuda-Kugo equation considered in the previous Section. In fact, since the kernel here is the difference of the terms already considered, the solution is very straightforward and we can immediately write down the result. The quark gap equation for timelike momentum reads

$$B_t(x)=\hat{B}(x)-I(x)+i\int_0^\infty d\lambda\frac{\rho(\lambda)}{\lambda-\mu^2}\sum_j\frac{X(x;m_j^2,\mu^2)\Theta(x-(\mu+m_j)^2)-X(x;m_j^2,\lambda)\Theta(x-(\lambda^{1/2}+m_j)^2)}{\left|1-\frac{dB_t(y)}{d(y^{1/2})}\right|_{y_j=m_j^2}} \tag{3.7}$$

$$\begin{aligned}
\hat{B}(x)&=m_0+\int_0^\infty dy\,\frac{B_s(y)}{y+B_s^2(y)}K(-x,y)\\
I(x)&=\int_0^\infty\frac{d\lambda}{\pi}\frac{\rho(\lambda)}{\lambda-\mu^2}P.\left[\int_0^{(\sqrt{x}-\mu)^2}dy\,X(x;y,\mu^2)\Theta(x-\mu^2)-\int_0^{(\sqrt{x}-\lambda)^2}dy\,X(x;y,\lambda)\Theta(x-\lambda)\right]\frac{B_t(y)}{y-B_t^2(y)}\\
K(-x,y)&=\int\frac{d\lambda}{\pi}\frac{\rho(\lambda)}{\lambda-\mu^2}K(-x,y,\mu^2)-K(-x,y,\lambda)\\
&=\int_0^\infty\frac{d\lambda}{\pi}\rho(\lambda)\frac{\mu^2-\lambda-\sqrt{(-x+y+\mu^2)^2+4xy}+\sqrt{(-x+y+\lambda)^2+4xy}}{-2x(\lambda-\mu^2)}
\end{aligned} \tag{3.8}$$

where the functions $K, X(a,b,c)$ have been already established in the previous Section.

Let us mention at this place that the one loop expression for analytical running coupling given by ρ_g (3.3) is inefficient to produce chiral symmetry breaking and the nonperturbative contribution which enhance the running coupling in the infrared has to be added. Clearly, the lack of nonperturbative information on the infrared contribution of the quark-gluon vertex that enhances $\alpha(q)$ behaviour in the infrared requires further investigation. Actually, there could be a strong enhancement of γ^μ form factor which is already expected from the one loop result [22] and from the recent SDE and lattice studies [23]. In this paper we do not to solve quark SDE with the set of recently known or improved approximations of analytical running coupling and we leave this matter for future more detailed investigation. Instead of this we illustrate the numerical solution for the case of simple approximation providing thus rather simple technical guidance for the more complicated investigation. For this purpose we take the running coupling spectral function $\rho_g = \rho$ to be dominated by a delta function centered at the QCD scale

$$\rho(\lambda) = \alpha^*(\Lambda^2 - \mu^2)\delta(\lambda - \Lambda^2) \tag{3.9}$$

where α^* is a dimensionless constant. Using (3.9) certainly represents rather crude approximation of the running coupling. However dealing with more realistic case, the procedure remains exactly the same. In our case, the appropriate running coupling does not has standard log behaviour but it behaves like the inverse of q^2 in ultraviolet regime. Recall that the rather similar approximation has been already considered by Higashijima [5] wherein the meaning of asymptotic freedom for dynamical chiral symmetry breaking has been recognized.

For purpose of completeness we write quark SDE explicitly here:

$$\begin{aligned}
B_s(x) &= m_0 + \frac{\alpha^*}{\pi}\int_0^\infty dy\, \frac{B_s(y)}{y + B_s^2(y)} K(x,y) \\
K(x,y) &= \left[K(x,y,\mu^2) - K(x,y,\Lambda^2)\right] \\
&= \frac{1}{2x}\{\mu^2 - \Lambda^2 - \sqrt{(x+y+\mu^2)^2 - 4xy} + \sqrt{(x+y+\Lambda^2)^2 - 4xy}\}
\end{aligned} \tag{3.10}$$

and the continuation on the analyticity cut reads

$$B_t(x) = \hat{B}(x) - I(x) + i\alpha^* \sum_j \frac{X(x; m_j^2, \mu^2)\Theta(x - (\mu + m_j)^2) - X(x; m_j^2, \Lambda^2)\Theta(x - (\Lambda + m_j)^2)}{\left|1 - \frac{dB_t(y)}{d(y^{1/2})}\right|_{y_j = m_j^2}}\,, \tag{3.11}$$

$$\begin{aligned}
\hat{B}(x) &= m_0 + \frac{\alpha^*}{\pi}\int_0^\infty dy\, \frac{B_s(y)}{y + B_s^2(y)} K(-x,y)\,, \\
I(x) &= \frac{\alpha^*}{\pi} P.\int_0^{(\sqrt{x}-\mu)^2} dy\, \frac{B_t(y)}{y - B_t^2(y)} X(x; y, \mu^2)\Theta(x - \mu^2) - \frac{\alpha^*}{\pi} P.\int_0^{(\sqrt{x}-\Lambda)^2} dy\, \frac{B_t(y)}{y - B_t^2(y)} X(x; y, \Lambda^2)\Theta(x - \Lambda^2)\,, \\
K(-x,y) &= K(-x,y,\mu^2) - K(-x,y,\Lambda^2) \\
&= \frac{1}{-2x}\{\mu^2 - \Lambda^2 - \sqrt{(-x+y+\mu^2)^2 + 4xy} + \sqrt{(-x+y+\Lambda^2)^2 + 4xy}\}\,.
\end{aligned} \tag{3.12}$$

The equations (3.11) have been solved numerically by the method of iterations, some useful details concerning the numerical treatment can be found in the Appendix.

Remind here known feature of QCD scaling: the constituent quark masses are of the same size as the QCD scale, i.e. $B(0) \simeq \Lambda$. In this paper, for any reason, we freely focus on a larger region of parameter space providing us the results for softer $B(0) < \Lambda$ and stronger $B(0) > \Lambda$ coupling case. We plot sample of numerical solutions in Fig. 3-6. For most of the solutions we fix the mass ratio to be $\Lambda^2/\mu^2 = 10$, the exception is explicitly mentioned. The critical value of the coupling has been identified $\alpha_c^* \simeq 3.8$, bellow that we do observe the trivial solution only. In Fig. 3 the numerical solution is plotted for the coupling strength $\alpha^* = 4.0$. Being rather close to the critical value α_c^*, the quark propagator has a one real pole at some point m which value is very closed to the infrared mass $B(0)$. Without any doubt, the constituent mass can be freely identified with the infrared mass or with the pole one. In Fig 4. the solution for $\alpha^* = 5$ is shown and we observe that propagator develops two real poles under the first branch point. The absorptive part of B is largely enhanced because the both differentiations $dB(m)/dm$ are not so far from 1 (see Eq. (3.11)). Such solution may or may not be an artifact of our approximation, here we simply mention its existence and point out that this is in agreement with already made observation in [11]. Increasing the coupling further, then the second pole vanishes at some point and we retain with the one real pole solution again. This situation is exhibited in Fig 5. . Note, the propagator becomes largely enhance bellow m, since the function B_t lies very closed to the diagonal

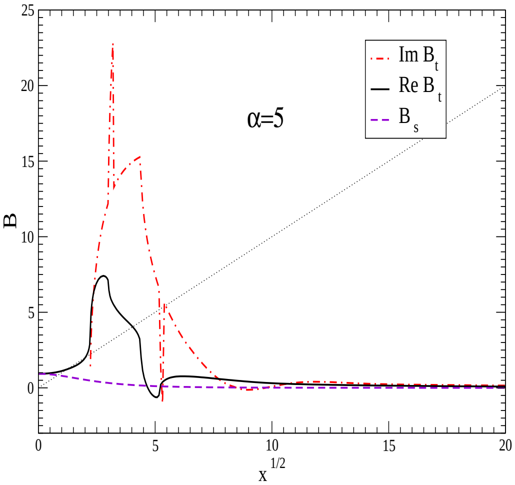

FIG. 4: Dynamical fermion mass for $\alpha^* = 5$ and $\mu/\Lambda = \sqrt{0.1}$.

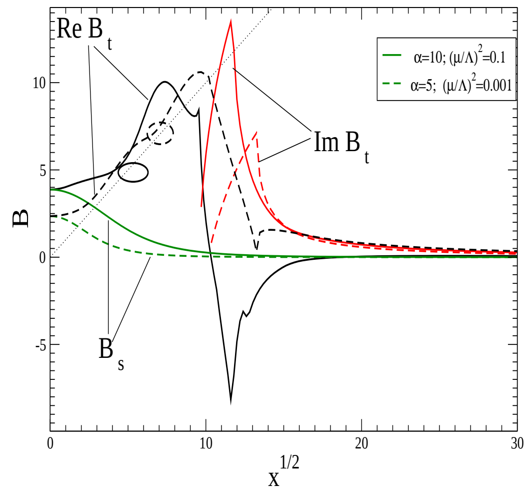

FIG. 5: Dynamical masses for 'super-strong' coupling as described in the text. Solid lines represent result for $\alpha^* = 10$ and $\mu/\Lambda = \sqrt{0.1}$, dashed line stands for $\alpha^* = 5$ and $\mu/\Lambda = \sqrt{0.001}$

of $\sqrt{p^2}, B_t$ graph. Contrary to the case of solutions with two poles, the imaginary part of B appears to be suppressed now, because of differentiation $B'(m) < 0$. In that case the pole mass largely differ from the infrared value $B(0)$, it is typically few times higher.

IV. DISCUSSION OF THE RESULTS

In this section we continue the discussion of the results and qualitatively compare with the already known results presented in literature. We also discussed the location of the poles in the complex plane and its relations with the violation of Lorentz invariance.

The asymptotic freedom ensure finiteness of of dynamically generated mass without the use of any artificial regularization scheme. The main result of our paper is the numerical observation of the fact that the *real pole is never absent* in translation invariance preserving calculation scheme. We have found strong evidence fort this statement by analyzing large parameter space of α^*, μ and Λ. Approaching the limit of vanishing boson mass μ , it seems to be true also for exactly boson massless case (see Fig. 5 for a small ratio $\mu^2/\lambda^2 = 0.001$), however we should note that the exact massless limit may not be well defined in approach presented here. Recall, the functional used in our equations is defined on the set of differentiable functions, while in contradiction, the differentiation $B'(m)$ may not exist in the exact massless case $\mu = 0$. In practise, the differentiation at the branch points is identified within a certain numerical uncertainty only, therefore we prefer to use nonzero μ in our numeric treatment.

Stating without proof, we suppose that true mass generation and the evidence of nontrivial pole of the fermion propagator go hand by hand with the property of asymptotic freedom (decreasing of effective coupling at ultraviolet), whilst the absence of real pole as happened to the propagator function in Fukuda-Kugo equation is artifact of improper calculation scheme (hard cutoff regularization). Further note, the numerical results obtained here qualitatively correspond with the results of the papers [11], where the same model was studied in the Minkowski space for the first time. At this place we discuss some common and some distinct features of solutions presented here and in the paper [11]. In both approaches the validity of Wick rotation is assumed. In other words, no (dynamically generated) complex singularities are assumed in the first and the third quadrant of p^2 complex plane. The main distinction is the analyticity assumption explicitly used in the paper [11], where it was assumed that the propagator is holomorphic in the whole p^2 complex plane up to a real positive semi-axis and that the propagator satisfy certain integral (generalized

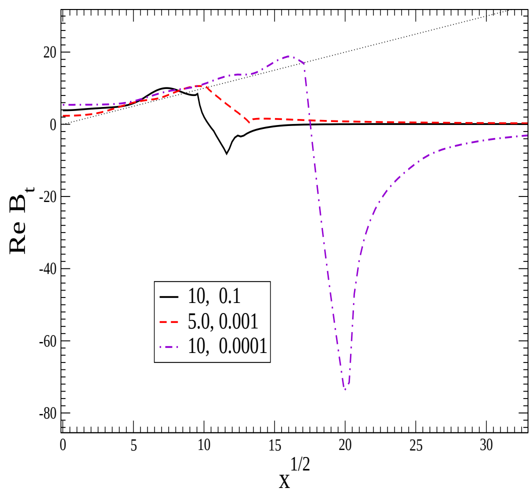

FIG. 6: Real part of the dynamical mass function for $x = p^2$. The lines shown represent the models characterized by couple of parameters α and the ratio μ^2/Λ^2.

spectral) representation. To this point, there is no similar assumption in the approach developed by Fukuda-Kugo and followed in presented paper. In the method presented here the correlator at timelike regime of fourmomenta is built on a base of the knowledge of its spacelike counter-partner. However, being possibly out of the domain of analyticity we should be aware that such continuation is by no mean unique. On the other hand , there is also reasonable quantitative agreement with [11], especially when we are not far form the critical coupling characterizing dynamical chiral symmetry breaking. Actually, depending on the coupling strength α^* the propagator develops one or two real poles with corresponding branch points providing their values are in an approximate agreement with [11].

In the literature there is a certain effort to guess the complex structure of QCD Green's function by the method of continuation of Euclidean result or by indirect reading from the phenomenological consequences which would follow form particularly assumed singularity structure of Greens functions. Complex conjugated singularities of quark and fermion propagator in planar strong coupling QED have been considered in the context of confinement [24] and in PT symmetric Quantum Field theory [25]. Additionally recent studies have modeled Euclidean space lattice data with propagators that have complex conjugate singularities [26, 27]. More phenomenologically, the meson bound states [28] and the parton distributions [29] has been calculated with a quark consisting of pairs of complex four momenta. In many, if not at all of these recent studies, the fact that complex singularity structure can affect the Lorentz invariance of the theory has been overlooked. Therefore, in what follow we mention the question of Wick rotation, location of complex poles and possible lost of Lorentz invariance in order to pay attention for.

Actually, increasing the coupling strength one can expect that apart of the real pole a new complex singularities appear. The area of real part of p^2 where we can expect new complex singularities in the propagator function is indicated by the ellipses in Fig. 6. In recent, we are not able to estimate the characteristic of the behaviour and/or the position of these complex singularities with good confidence. Instead of this, we would like to present the argument which largely enforces our believe in the validity of Wick rotation in a case of full realistic solution of Schwinger-Dyson equations. Recall at this place, there is known relation between location of complex singularities of Greens functions and the Lorentz invariance of the theory. Indeed, its more than 30 years known that complex singularities located simultaneously on both side of the real axis of p^2 automatically generates Lorentz violating peaces in the propagator itself [30]. In this case the naive Wick rotation invalidates and one has to account the complex singularities in a suitable manner. Actually, assuming the complex poles (with non vanishing Im part) one can exhibit the existence of Lorentz violating pieces originally not expected in. The evaluation of the appropriate Feynman integral has been exhibited for the case of complex conjugated poles in [30] (some prefactor has been corrected in the paper [31], the actual evaluation has been performed for scalar field content only, the extension to the loop integral with internal fermion links is rather straightforward) due to the rather different reasons. The generalization to more general locations of singularities were already discussed in [30]. From these arguments it follows: if the true exact solution respect the Lorentz invariance of the theory then selfconsistence of the Schwinger-Dyson equation solution for quark propagator can not involve complex poles on the both sides of the real axis of the square of the fourmomentum. In Lorentz invariant theory, the Wick rotation is indirectly justified in this way.

V. CONCLUSION

The model we have described , is of course not true QCD and hence we do not claim that the fermion propagator we have studied here is truly representative of the singularity structure of the quark propagator. In a more realistic models, which include the dressed vertex unction and/or higher order skeleton graphs, one expects that the details of the propagator structure would be different. It should also incorporate the aspect of asymptotic freedom in a more proper way. However, our study demonstrates that singularity structure of the quark propagator is very likely dominated by the real singularity and we expect a quantitative but not a qualitative changes when the approximations improve. We discuss possible singularity structure in the entire complex plane of momenta. We argue, in the Lorentz covariant formulation of QCD (and any QFT) the singularities of the propagators (and of integral kernel of gap equations at all) must appear only on the one side -upper or down- separately for the left and the right half complex plane. Otherwise we necessarily sacrify Lorentz invariance of the theory. This argument strongly support the validity of Wick rotations in Lorentz invariant theory.

VI. ACKNOWLEDGMENTS

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APPENDIX A: ANGULAR INTEGRATION

The following integral:

$$\int_{-1}^{1} dz \frac{\sqrt{1-z^2}}{z-a} = \pi(-a+\sqrt{a^2-1})\,, \tag{A1}$$

is used in order to perform the angular integration in SDEs.

APPENDIX B: PRINCIPLE VALUE INTEGRATION

1. I for Fukuda-Kugo equation

SDE for timelike momentum (2.9) represents the complex inhomogeneous integral equation with singular kernel. In this appendix we describe some details how to numerically deal with. The integral to be evaluated reads

$$I(x) = \frac{3C}{4} P. \int_0^{(\sqrt{x}-\mu)^2} dy \, \frac{B_t(y)}{y - B_t^2(y)} X(x;y,\mu^2)\Theta(x-\mu^2) \tag{B1}$$

noting that the other terms in the SDE represent regular integrals with a smooth kernel.

Let us assume that we have made a good guess of the value of the pole mass m, then it is convenient to write down the expression (B1) separately for various regime of momenta term I equivalently as follows

$$I(x) = 0 \qquad\qquad x < \mu^2 \tag{B2}$$

The function $B(y)$ is real bellow the thresholds and the kernel is regular for x bellow the threshold, hence there is no need to denote $P.$ in front of integral since $y < m^2$ for $\sqrt(x) < m + \mu$. At the point $\sqrt(x) = m + \mu$ the kernel singularity is suppressed by vanishing function X, thus we can safely write

$$I(x) = \frac{3C}{4} \int_0^{(\sqrt{x}-\mu)^2} dy \, \frac{B_t(y)}{y - B_t^2(y)} X(x;y,\mu^2) \qquad\qquad \mu^2 < x < (\mu+m)^2 \tag{B3}$$

For a larger x we necessarily cross the singularity in the kernel, however $B(y)$ remains real to the threshold and we consider this regime separately:

$$I(x)=\frac{3C}{4}P.\int_0^{(\sqrt{x}-\mu)^2}dy\,\frac{B_t(y)}{y-B_t^2(y)}X(x;y,\mu^2)\,, \qquad (\mu+m)^2<x<(2\mu+m)^2 \tag{B4}$$

Increasing x the kernel become complex and we divide the integral to the P. value integration over the real B and to the regular integration over the regular kernel with complex B:

$$\begin{aligned}
I(x) &= \frac{3C}{4}P.\int_0^{(m+\mu)^2}dy\,\frac{B_t(y)}{y-B_t^2(y)}X(x;y,\mu^2)\\
&+ \frac{3C}{4}\int_{(m+\mu)^2}^{(\sqrt{x}+\mu)^2}dyX(x;y,\mu^2)\left[\Sigma_R\frac{y-\Sigma_R^2-\Sigma_I^2}{\left(y-\Sigma_R^2+\Sigma_I^2\right)^2+4\Sigma_R^2\Sigma_I^2}+i\,\Sigma_I\frac{y+\Sigma_R^2+\Sigma_I^2}{\left(y-\Sigma_R^2+\Sigma_I^2\right)^2+4\Sigma_R^2\Sigma_I^2}\right],\\
&\text{for}\qquad x>(2\mu+m)^2\,,
\end{aligned}\tag{B5}$$

where we have used following shorthand notation

$$\Sigma_R=\mathbf{Re}B_t(y);\;\;\Sigma_I=\mathbf{Im}B_t(y). \tag{B6}$$

To avoid some unwanted numerical fluctuations which usually stem from asymmetric distribution of mesh points when P. integration is numerically performed, we use a standard trick. Consider for this purpose the third of considered momentum regime where x runs over the interval $((\mu+m)^2,(2\mu+m)^2)$ (see (B4)) . In our numerical treatment the integral is replaced by a discrete sum with the appropriate (Gaussian) weights, i.e.

$$I(x)=\frac{3C}{4}P.\int_0^{(\sqrt{x}-\mu)^2}dy\,\frac{B_t(y)}{y-B_t^2(y)}X(x;y,\mu^2)\;\rightarrow\frac{3C}{4}\sum_j w(y_j)\,\frac{B_t(y_j)}{y_j-B_t^2(y_j)}X(x;y_j,\mu^2)\,. \tag{B7}$$

The numerical fluctuations are subtracted by the following trick

$$\begin{aligned}
&\frac{3C}{4}\sum_j w(y_j)\,\frac{B_t(y_j)}{y_j-B_t^2(y_j)}X(x;y_j,\mu^2)\\
&=\frac{3C}{4}\left\{\sum_j w(y_j)\left[\frac{B_t(y_j)}{y_j-B_t^2(y_j)}X(x;y_j,\mu^2)-\frac{mX(x;m^2,\mu)}{y_j-m^2}\right]+X(x;m^2,\mu^2)\log|\frac{(\sqrt{x}-\mu)^2-m^2}{-m^2}|\right\},
\end{aligned}\tag{B8}$$

where the later two terms vanishes in the exact continuum limit. We have found this approach is very stable and the pole mass m can be identified after few runs of the iteration program. In order to achieve better stability, the actual search of the root of equation $B(x)-x=0$ has been performed by hand for each solution. After making a few iterations in m_j then the achieved numerical accuracy of our search is estimated by $\Delta m\simeq step/m$ where 'step' means the difference of two neighboring points at vicinity of m. Likewise in the case of SDE in Euclidean space, the integrals were cut by the ultraviolet cutoff.

2. Kernel for QCD like equation

Generalizing to the case of Eq. (3.11) is rather straightforward. The principal value integration is treated in the same fashion as in the case of Fukuda-Kugo equation. Now the relevant term reads

$$I(x)=I_\mu(x)-I_\Lambda(x)\,, \tag{B9}$$

where I_μ and I_Λ are the integrals considered previously, but where μ is replaced by QCD scale Λ_{QCD} in the later case. Contrary to the previous case the integral is insensitive to the value of upper boundary Λ_H, when it $\Lambda_H>\Lambda_{QCD}$

For completeness we list the function I_Λ bellow

$$I_\Lambda(x)\;=\;0, \qquad\qquad x<\Lambda^2$$

$$
\begin{aligned}
I_\Lambda(x) &= \frac{3C}{4}\int_0^{(\sqrt{x}-\Lambda)^2} dy\,\frac{B_t(y)}{y-B_t^2(y)}X(x;y,\Lambda^2)\,, \qquad\qquad \Lambda^2<x<(\Lambda+m)^2\\
I_\Lambda(x) &= \frac{3C}{4}P.\int_0^{(\sqrt{x}-\Lambda)^2} dy\,\frac{B_t(y)}{y-B_t^2(y)}X(x;y,\Lambda^2)\,, \qquad\qquad (\Lambda+m)^2<x<(\mu+\Lambda+m)^2\\
I_\Lambda(x) &= \frac{3C}{4}P.\int_0^{(m+\mu)^2} dy\,\frac{B_t(y)}{y-B_t^2(y)}X(x;y,\mu^2)\\
&+ \frac{3C}{4}\int_{(m+\mu)^2}^{(\sqrt{x}-\Lambda)^2} dyX(x;y,\Lambda^2)\left[\Sigma_R\frac{y-\Sigma_R^2-\Sigma_I^2}{\left(y-\Sigma_R^2+\Sigma_I^2\right)^2+4\Sigma_R^2\Sigma_I^2}+i\,\Sigma_I\frac{y+\Sigma_R^2+\Sigma_I^2}{\left(y-\Sigma_R^2+\Sigma_I^2\right)^2+4\Sigma_R^2\Sigma_I^2}\right],\\
&\text{for} \qquad\qquad x>(\mu+\Lambda+m)^2 \qquad\qquad \text{(B10)}
\end{aligned}
$$

The treatment of principal value integrals is the same as in the previous case of Fukuda-Kugo equation.

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[1] D. J. Gross and F. Wilczek, Phys. Rev. Lett. **30**, 1343 (1973); H. D. Politzer, *ibid*, **30**, 1346 (1973); G. t'Hooft at the conference on Yang-Mills fields (Marseille, France, 1972); S. Weinberg, Phys. Rev. Lett. **31**,494 (1973); D. Gross, Phys. Rev. **D8**,3633 (1973).

[2] Mandelstam S., Phys. Rev. **D20**, 3223 (1979); Brown N., Pennington M. R., Phys. Rev. **D39**; 2723 (1989); L. von Smekal, A. hauck and R. Alkofer, Ann. Phys. **267**; 1 (1998); Atkinson D., Bloch J.C.R., Phys. Rev. **D58**, 094036n (1998); Atkinson D., Bloch J.C.R., Mod. Phys. Lett, **A13** 1055 (1998);

[3] Cornwall J.M., Phys. Rev. **D26**, 1453 (1982).

[4] C. S. Fischer, J. Phys.**G32**, R253 (2006).

[5] Higashijima K., Phys. Rev. **D29**, 1228 (1984).

[6] J. Braun, H. Gies, JHEP **0606**, 024 (2006);

[7] M. Kurachi, R. Shrock, JHEP 0612, 034 (2006).

[8] C. S. Fischer, PhD-thesis, Univ. of Tuebingen. Nov 2002, hep-ph/0304233; C. S. Fischer, P. Watson, W. Cassing, Phys. Rev. **D72**,094025 (2005);

[9] Roberts C. D. and Williams A. G., Prog. Part. Nucl. phys. Vol. 33 Oxford, pergamon Press 1994.

[10] A. Blumhofer and J. Manus, Nucl. Phys. **B515**, 522 (1998);

[11] V. Sauli, J. Adam, Jr. , P. Bicudo ArXive: hep-ph/0607196; V. Sauli, hep-ph/0607260.

[12] Fukuda R., Kugo T. , Nucl. Phys. B 117, 250 (1976).

[13] D.V. Shirkov, I.L. Solovtsov, Phys. Rev. Lett. **79**, 1209 (1997); K. A. Milton, I. L. Solovtsov, Phys. Rev. **D55**, 5295 (1997); K. A. Milton, O. P. Solovtsova, Phys. Rev. **D57**, 5402 (1998).

[14] G. Grunberg, Phys. Rev. , **D29**, 2315 (1984).

[15] A.C. Aguilar, A.V. Nesterenko, J. Papavassiliou, J. Phys. **G31**,997 (2005); A.V. Nesterenko, J. Papavassiliou, Nucl. Phys. Proc. Suppl. **152**, 47 (2006).

[16] G. M. Prosperi, M. Raciti, C. Simolo,hep-ph/0607209.

[17] C. Itzykson, J. B. Zuber, Quantum Field Theory.

[18] K. Johnson, M. baker and R. Willey, Phys. Rev. **136** B1111 (1964); K. Johnson and M. Baker, Phys. Rev.**D8**, 1110 (1974); T. Maskawa and H.Nakajima, prog. Theor. Phys. **52**, 1326 (1974); **54**, 860 (1975).

[19] M. J. Lavelle and M. Schaden, Phys. Lett. B **208**, 297 (1988); F. V. Gubarev, L. Stodolsky and V. I. Zakharov, Phys. Rev. Lett. **86**, 2220 (2001); H. Verschelde, K. Knecht, K. Van Acoleyen and M. Vanderkelen, Phys. Lett. B **516**, 307 (2001); K. I. Kondo, T. Murakami, T. Shinohara and T. Imai, Phys. Rev. D **65**, 085034 (2002); U. Ellwanger and N. Wschebor, Int. J. Mod. Phys. A **18**, 1595 (2003); D. Dudal, H. Verschelde, J. A. Gracey, V. E. R. Lemes, M. S. Sarandy, R. F. Sobreiro and S. P. Sorella, JHEP **0401**, 044 (2004); A. C. Aguilar, A. A. Natale and P. S. Rodrigues da Silva, Phys. Rev. Lett. **90**, 152001 (2003); R. E. Browne and J. A. Gracey, Phys. Lett. B **597**, 368 (2004); J. A. Gracey, Eur. Phys. J. C **39**, 61 (2005); K. Amemiya and H. Suganuma, Phys. Rev. D **60**, 114509 (1999); C. Alexandrou, P. de Forcrand and E. Follana, Phys. Rev. D **65**, 114508 (2002); E. Ruiz Arriola, P. O. Bowman and W. Broniowski, Phys. Rev. D **70**, 097505 (2004); T. Suzuki, K. Ishiguro, Y. Mori and T. Sekido, Phys. Rev. Lett. **94**, 132001 (2005); M. N. Chernodub *et al.*, Phys. Rev. D **72**, 074505 (2005).

[20] G. Parisi and R. Petronzio, Phys. Lett. B **94**, 51 (1980); J. Greensite and M. B. Halpern, Nucl. Phys. B **271**, 379 (1986); J. H. Field, Phys. Rev. D **66**, 013013 (2002); A. P. Szczepaniak and E. S. Swanson, Phys. Lett. B **577**, 61 (2003); E.G.S. Luna, A.A. Natale, Phys. Rev. **D73**,074019 (2006); E.G.S. Luna, Phys. Lett. **B641**,171 (2006).

[21] S. J. Brodsky, J. R. Pelaez, N. Toumbas, Phys. Rev. **D60**,037501 (1999).

[22] A. I. Davydychev, P. Osland , L. Saks, Phys. Rev. **D63**, 014022 (2001); Nucl. Phys. Proc. Suppl. **89**,277 (2000).

[23] M. S. Bhagwat, A. Hoell, A. Krassnigg, C. D. Roberts, P.C. Tandy, Phys. Rev. **C70**,035205 (2004); J. Skullerud, A. Kizilersu, JHEP **0209**,013 (2002). **dodat reference**

[24] D. Atkinson and D. W. Blatt, Nucl. Phys. B**151**,342 (1979); C. J. Burden, C.D. Roberts and A. G. Williams, Phys. Lett. B**285**, 347 (1992); G. Krein, C.D. Roberts and A. G. Williams, Int. J. Mod. Phys. A**7**, 5607 (1992); U. Habel, R. Konning,

H. G. Reusch, M. Stingl, S. Wigard, Z. Phys. A**336**,423 (1990); M. Stingl, Z. Phys. A**353**,423 (1996); P. Maris, Phys. Rev. D**52**,6087 (1995); V. N. Gribov, Eur. Phys. J. C**10**,91 (1999).

[25] C. M. Bender, Sebastian F. Brandt, J-H Chen, Q. Wang, Phys. Rev. D**71**; 025014 (2005).

[26] M. S. Bhagwat, M. A. Pichowsky, C. D. Roberts, P. C. Tandy, Phys. Rev. C**68**, 015203 (2003);

[27] R. Alkofer, W. Detmold, C. S. Fischer, P. Maris Comments: 42 pages, 16 figures, revtex; version to be published in Phys Rev D Journal-ref: Phys.Rev. D70 (2004) 014014

[28] M. Bhagwat, M. A. Pichowsky, P. C. Tandy, Phys. Rev. D**67**,054019 (2003).

[29] B. C. Tiburzi, W. Detmold, G. A. Miller, Phys. Rev. D**68**,073002 (2003).

[30] A.M. Gleeson, R.J. Moore, H. Rechenberg, E.C.G. Sudarshan, Phys. Rev. **D4** ,2242 (1971); N. Nakanishi, Phys. Rev. **D3**,811 (1971).

[31] N. Nakanishi, hep-th/0609206.